3) A particle moves according to a law of motion $\boxed{s = f(t)}$, $t \geq 0$, where t is measured in seconds and s is measured in feet.

$f(t) = t^3 - 12t^2 + 36t$

a) Find the velocity at time t. $v(t) = 3t^2 - 24t + 36$

b) What is the velocity after 3s? $v(3) = 3(3)^2 - 24(3) + 36 = -9 \text{ ft/sec}$

c) When is the particle at rest? $0 = 3t^2 - 24t + 36 = 3(t^2 - 8t + 12)$

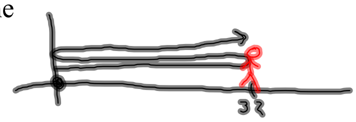
$3(t-6)(t-2)$

2sec, 6sec

d) When is the particle moving in the positive direction? $(0,2) \cup (6, \infty)$

e) Find the total distance traveled during the first 8 sec. $s(2)-s(0)$ 32 $\quad s(2)-s(6)$ 32 $\quad s(8)-s(6)$ 32 $\quad = 96 \text{ ft}$

f) Draw a diagram to illustrate the motion of the particle.



7) The position function of a particle is given by

$s = t^3 - 4.5t^2 - 7t, \quad t \geq 0.$

When does the particle reach a velocity of 5 m/s?

$v(t) = 3t^2 - 9t - 7$

$5 = 3t^2 - 9t - 7$

$0 = 3t^2 - 9t - 12$

$0 = t^2 - 3t - 4$

$(t-4)(t+1)$

-1 sec, 4 sec

8) If a ball is given a push so that it has an initial velocity of 5 m/s down a certain inclined plane, then the distance it has rolled after t seconds is $s = 5t + 3t^2$.

a) Find the velocity after 2 sec.

b) How long does it take for the velocity to reach 35 m/s?

Oct 22-2:25 PM

9) If a stone is thrown vertically upward from the surface of the moon with a velocity of 10 m/s, its height (in meters) after t seconds is $h = 10t - 0.83t^2$.

$s = \frac{1}{2}at^2 + v_0t + s_0$

a) What is the velocity of the stone after 3 s?

$\boxed{V}$ m/s $= 10 - 1.66t$ /s

$v(3) = 10 - 1.66(3)$

$= 5.02$ m/s

b) What is the velocity of the stone after it has $\underbrace{\text{risen}}_{\uparrow}$ 25 m?

$25 = 10t - 0.83t^2$

$t = \boxed{3.54, 8.51}$ sec

$v(3.54) = 10 - 1.66(3.54)$

$= 4.12$ m/s

Oct 22-2:28 PM

10) If a ball is thrown vertically upward with a velocity of 80 ft/sec, then its height after t seconds is $s = 80t - 16t^2$.

a) What is the maximum height reached by the ball?

$v = 80 - 32t$

$0 = 80 - 32t$

$32t = 80$

$t = 2.5$ sec

$s = 80(2.5) - 16(2.5)^2$

$= 100$ ft

b) What is the velocity of the ball when it is 96 ft above the ground on its way up? On its way down?

$96 = 80(t) - 16t^2$

$t = \underline{\underline{2}}, 3$ sec

$v(2) = 80 - 32(2)$

$= 16$ ft/sec

$v(3) = -16$ ft/sec

Oct 22-2:29 PM

14) A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 cm/s. Find the rate at which the area within the circle is increasing after

a) 1 sec, $A'(1) = 7200\pi \ \text{cm}^2/\text{sec}$

b) 3 sec,

c) 5 sec.

$\text{cm}^2 \quad A(r) = \pi r^2 \quad$ cm, $\frac{\text{cm}}{\text{sec}}$

$r(1) = 60$

$r(2) = 120$

$r(3) = 180$

$\text{cm}^2 \quad A(t) = \pi(60t)^2$

$= 3600\pi t^2$

$A'(t) = 7200\pi t$ $\quad \text{cm}^2/\text{sec}$

$A''(t) = 7200\pi$ $\quad \text{cm}^2/\text{sec}^2$

What can you conclude?

$A(r) = \pi r^2$

$\text{cm}^2/\text{cm} \quad A'(r) = 2\pi r$

Oct 22-2:36 PM

15) A spherical balloon is being inflated. Find the rate of increase of the surface area ($S = 4\pi r^2$) with respect to the radius r when r is

a) 1 ft,

b) 2 ft,

c) 3 ft.

What conclusions can you make?

Oct 22-3:00 PM

18) If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then Torricelli's Law give the volume V of water remaining in the tank after t minutes as $V = 5000\left(1 - \frac{t}{40}\right)^2$, [0,40].

Find the rate at which water is draining from the tank after

a) 5 min,

b) 10 min,

c) 20 min,

d) 40 min.

$$V = 5000\left(\frac{40-t}{40}\right)^2 \left(\frac{40-t}{40}\right)$$

$$V = \overset{25}{\cancel{5000}}\left(\frac{1600 - 80t + t^2}{\underset{8}{\cancel{1600}}}\right)$$

$$V = 5000 - 250t + \frac{25}{8}t^2$$

$$\frac{dV}{dt} = -250 + \frac{25}{4}t$$

At what time is water flowing out the fastest?
slowest?

Summarize your findings.

Oct 22-3:07 PM

19) The quantity of charge Q in coulombs (C) that has passed through a point in a wire up to time t (measured in seconds) is given by $Q(t) = t^3 - 2t^2 + 6t + 2$. Find the current when :

a) $t = 0.5$ sec,

b) $t = 1$ sec,

The unit of current is an ampere (1 A = 1 C/sec).
At what time is the current lowest?

Oct 23-7:30 AM

20) Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is $F = \frac{gmM}{r^2}$ where g is the gravitation constant and r is the distance between the bodies.

a) Find $\frac{dF}{dr}$ and explain its meaning. What does the minus sign indicate?

b) Suppose it is known that the Earth attracts an object with a force that decreases at the rate of 2 N/km when r = 20,000 km. How fast does this force change when r = 10,000 km.

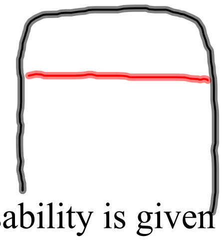
Oct 23-7:34 AM

21) Boyle's Law states that when a sample of gas is compressed at a constant temperature, the product of the pressure and the volume remains constant: $PV = C$. $V = \frac{c}{P} = cP^{-1}$

a) Find the rate of change of volume with respect to pressure.

$$\downarrow V' = -cP^{-2} = \frac{-c}{P^2} \uparrow$$

b) A sample of gas in a container at low pressure and is steadily compressed at constant temperature for 10 minutes. Is the volume decreasing more rapidly at the beginning or the end of the 10 minutes? Explain.



c) Prove that the isothermal compressability is given by: $\beta = \frac{1}{P}$

$\boxed{\text{Skip}}$

33) The gas law for an ideal gas at absolute temperature T (in kelvins), pressure P (in atmospheres), and volume V (in liters) is $PV = nRT$, where n is the number of moles of the gas and $R = 0.0821$ is the gas constant. Suppose that, at a certain instant, $P = 8.0$ atm and is increasing at a rate of 0.10 atm/min and $V = 10$ L and is decreasing at a rate of 0.15 L/min. Find the rate of change of T with respect to time at that instant if $n = 10$ mol.

35) In the study of ecosystems, *predator-prey* models are often used to study the interaction between species. Consider populations of tundra wolves, given by $W(t)$, and caribou, given by C(t), in northern Canada. The interaction has been modeled by the equations

$$\frac{dC}{dt} = aC - bCW \qquad \frac{dW}{dt} = -cW + dCW$$

a) What values of $\frac{dC}{dt}$ and $\frac{dW}{dt}$ correspond to stable populations?

b) How would the statement "The caribou go extinct" be represented mathematically?

c) Suppose that $a = 0.05$, $b = 0.001$, $c = 0.05$, and $d = 0.0001$. Find all population pairs (C,W) that lead to stable populations. According to this model, is it possible for the two species to live in balance or will one or both species become extinct?

Oct 23-7:52 AM